Name:

SM223 – Calculus III with Optimization Fall 2017
Assoc. Prof. Nelson Uhan

Exam 3 – 27 October 2017

Instructions

- You have until the end of the class period to complete this exam.
- You may use a calculator.
- You may not consult any other outside materials (e.g. notes, textbooks, homework).
- **Show all your work.** Your answers should be legible and clearly labeled. It is your responsibility to make sure that I understand what you are doing. You will be awarded partial credit if your work merits it.
- Keep this booklet intact.
- **Do not discuss the contents of this exam with any midshipmen until it is returned to you.**

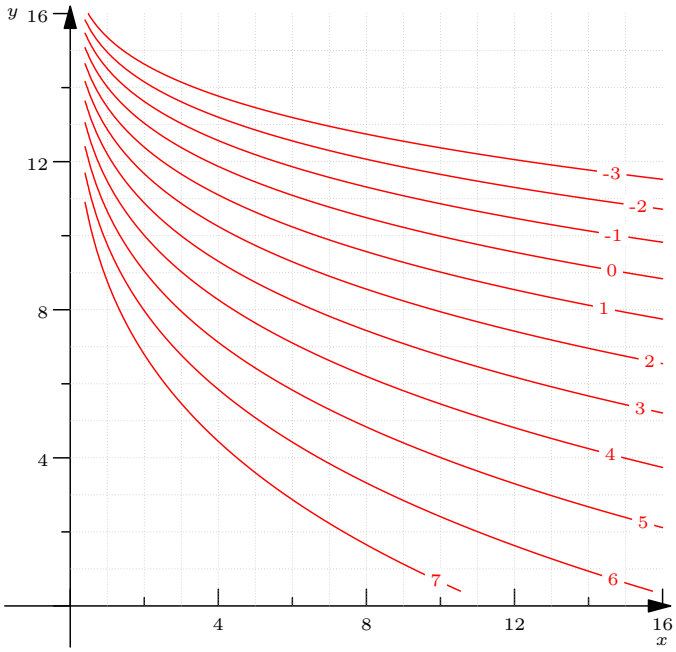
Problem	Weight	Score
1	$1\frac{1}{2}$	
2	1	
3	1	
4	1	
5	$1\frac{1}{2}$	
6	$1\frac{1}{2}$	
7	1	
8	$1\frac{1}{2}$	
Total		/ 100

Problem 1. Let $f(x, y) = e^{x-y}$. Find an equation of the tangent plane to the surface $z = f(x, y)$ at $(2, 2, 1)$. Use your equation to approximate the value of $f(2.02, 1.99)$.

Problem 2. Wine production W in a given year depends on the average temperature T and the annual rainfall R. Scientists estimate that the average temperature is rising at a rate of 0.15°C/year and rainfall is decreasing at a rate of 0.1 cm/year. They also estimate that at current levels, $\partial W/\partial T = -2$ and $\partial W/\partial R = 8$.

Estimate the current rate of change of wine production.

For Problems 3-4, consider the contour map for the function f shown below.



Problem 3. Estimate the directional derivative of f at $(4, 7)$ in the direction $\langle -1, 1 \rangle$. Briefly explain.

Problem 4. Draw the direction of the gradient at $(10, 4)$. Briefly explain.

Problem 5. Let $f(x, y) = x^2 \ln y$. Find the rate of change of f from the point $P(3, 1)$ towards the point $Q(-2, 13)$.

Problem 6. Find parametric equations of the line normal to the surface $x = y^2 + z^2 + 1$ at $(3, 1, -1)$.

Problem 7. Let $f(x, y) = x^2 + y^2 + xy + y$. The only critical point of f is $(1/3, -2/3)$. Find the local minimum values, the local maximum values, and the saddle points of f.

Problem 8. Find all of the critical points of $f(x, y) = x^2 + y^4 + 2xy$.